647510 **BSCmaMJ206x-3C** **Seat No :________**

B.Sc. Semester - 3 (NEP-2020) Examination
OCT/NOV-2025 (AS PER SYLLABUS YEAR JUNE 2024)
MATH: Advanced Calculus -1 (Major-6)

Time: 2:00 Hours **Marks:50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two. (10)

(1) Define: Divergence of a vector function. If $\bar{f}$ and $\bar{g}$ are vector functions defined on the domain D, whose first order partial derivatives exist then prove that $div(\bar{f} \times \bar{g}) = \bar{g} \cdot curl\bar{f} - \bar{f} \cdot curl\bar{g}$.

(2) Define: Irrotational function. If $\bar{f} = (x^3, y^3, z^3)$ then show that $\bar{f}$ is irrotational function and $grad(div\bar{f}) = 6\bar{r}$.

(3) Define: Laplace equation. Prove that $H = e^{-\lambda x}(c_1 sin\lambda y + c_2 cos\lambda y)$ satisfies the Laplace equation, where λ, c_1, c_2 are constants.

Que.2 Answer any two. (10)

(1) Evaluate $\iint_R (x^2 + y^2)\, dxdy$, where R is a square bounded by lines $y = x,\ y = -x,\ x - y = 2, x + y = 2$.

(2) Prove that: $\iiint_R (x^2 + y^2 + z^2)^5 dxdydz = \frac{4\pi a^{13}}{13}$, where R is $x^2 + y^2 + z^2 = a^2$.

(3) Change the order of integration in $\int_0^1 \int_{\frac{x^2}{a}}^{2a-x} f(x, y)dxdy$ and hence evaluate it.

Que.3 Answer any two. (10)

(1) Evaluate: $\oint_c \bar{u}_t ds$, where $\bar{u} = y^2\hat{\imath} + xy\hat{\jmath}$ and c is a square with vertices $(1,1), (-1,1), (-1,-1)$ and $(1,-1)$.

(2) State and prove Green's theorem in plane.

(3) Verify the divergence theorem for $\bar{V} = x\hat{\imath} + y\hat{\imath} + z\hat{k}$ and S is a sphere $x^2 + y^2 + z^2 = 1$.

Que.4 Answer any two. (10)

(1) Prove that : $\beta(p, q) = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx \,; p > 0, q > 0$.

(2) Define: Gamma function. Prove: $\int_0^{\frac{\pi}{2}} sin^m\theta cos^n\theta d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$.

(3) Prove that $\int_0^1 \frac{x^5}{\sqrt{1-x^4}} dx = \frac{\pi}{8}$.

Que.5 Answer any two. (10)

(1) If $\bar{f}$ and $\bar{g}$ are irrotational functions on D then show that $\bar{f} \times \bar{g}$ is a solenoidal function.

(2) Evaluate: $\oint_c e^{-x} sinydx + e^{-x} cosydy$; where c is a rectangle with vertices $(0,0), \left(0, \frac{\pi}{2}\right), \left(\pi, \frac{\pi}{2}\right)$ and $(\pi, 0)$.

(3) Prove: $\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$; $p > 0, q > 0$.

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